Exercise 1. *Assume that A is the set of integers divisible by 4. Similarly assume that B and C are the sets of integers divisible by 9 and 10, respectively. What is in $A \cap B \cap C$?*

Answer 1. *The intersection of these sets is the least common multiple of 4, 9 and 10. Thus, $A \cap B \cap C = \{n \in \mathbb{N} \mid n \text{ is divisible by } 180\}$.*

Exercise 2. *Is $P(A - B)$ always equal to $P(A) - P(B)$? Is it ever equal to $P(A) - P(B)$?*

Answer 2. *At first glance, it appears that these two sets should always be equal. However, note that given some set Z, $\emptyset \subseteq P(Z)$. So, subtracting two power sets would remove this element, and thus even though $\emptyset \subseteq P(A - B)$, we also have $\emptyset \nsubseteq P(A) - P(A)$, and thus $P(A - B) \neq P(A) - P(B)$.*

Exercise 3. *Define the symmetric difference $A + B$ of sets A and B to be the set $(A - B) \cup (B - A)$. Show that $A \cap (B + C) = (A \cap B) + (A \cap C)$.*

Answer 3. *Since we have that $A + B = (A - B) \cup (B - A)$, we can write*

$$A \cap (B + C) = A \cap ([B - C] \cup [C - B]).$$

By distributive laws, this becomes

$$(A \cap [B - C]) \cup (A \cap [C - B]).$$

Now we need an additional fact, that given sets X, Y, and Z, $X \cap (Y - Z) = (X \cap Y) - (X \cap Z)$. For this we compose the following proof:

Let there be some $x \in X \cap (Y - Z)$. Then, $x \in X$ and also $x \in Y - Z$. Because $X \cap Y \subseteq X$, we know that $x \in X \cap Y$. Likewise, since $x \notin Z$, and $X \cup Z \subseteq Z$, we see that $x \notin X \cup Z$ also. So, we have that $x \in (X \cap Y) - (X \cap Z)$, and so $X \cap (Y - Z) \subseteq (X \cap Y) - (X \cap Z)$. To prove the logical converse, assume there exists some $x \in (X \cap Y) - (X \cap Z)$. Then, $x \in (X \cap Y)$ but also $x \notin (X \cap Z)$. Because x must exist in the intersection between X and Y, but cannot exist in the intersection between X and Z, we must have that $x \in Y - Z$. Similarly, since $x \in X \cap Y$ and since $X \cap Y \subseteq X$, we have that $x \in X$. These last two statements imply that $x \in X \cap (Y - Z)$, and so we can say that $X \cap (Y - Z) = (X \cap Y) - (X \cap Z)$.

Returning to the original proof, we can now show that $(A \cap [B - C]) \cup (A \cap [C - B]) = ([A \cap B] - [A \cap C]) \cup ([A \cap C] - [A \cap B])$. Notice that by definition of the symmetric difference, this can be rewritten as $(A \cap B) + (A \cap C)$, which is what was to be shown. Therefore, $A \cap (B + C) = (A \cap B) + (A \cap C)$.